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Preface

from President of IndoMS for Proceeding of IICMA 2009:

First of all, I would like to pray for God for His mercy so that we could finish the Proceeding of IICMA 2009 (IndoMS International Conference on Mathematics and its Applications 2009) hold on October 12^{th} - 13^{th} , 2009 at the Departement of Mathematics Gadjah Mada University Yogyakarta Indonesia. On behalf of the IndoMS (Indonesian Mathematical Society), I would like to say congratulation to all authors in the proceeding.

IndoMS or formerly known as “Himpunan Matematika Indonesia” is a forum for mathematicians and users of mathematics as well as people who have interest in enhancing mathematics in Indonesia. The Society is a scientific, nonprofit, non-governmental and professional organization. It was established on July 15^{th}, 1976 in Bandung, West Java. The objectives of the Society are to enhance and extend mathematical knowledge, extend education in the Mathematical sciences, and to increase the role of mathematics in Indonesia. In 2009 IndoMS has 1.151 members consisting of university teachers, mathematicians, statisticians and mathematics-education researchers from 30 Indonesian universities, and school teachers from elementary and high schools. IndoMS has established 8 provincial officers to stimulate and enhance mathematical activities in the country. The branches are branch Special Territory of Yogyakarta and Central Java, branch Banten, Special Territory of Jakarta, and West Java, branch East Java, branch South and West Sulawesi, branch South Kalimantan, branch South Sumatera, branch Nanggro Aceh Darussalam and South Sumatra, and branch East Nusa Tenggara. Since 1976, IndoMS has already 14 times organized National Conference in Mathematics and National Congress. The next National Conference in Mathematics and National Congress will be held in Manado State University, North Sulawesi on June 30 – July 3, 2010. Since 2006, IndoMS also has already 3 times organized National Conference in Mathematics Education. The next National Conference in Mathematics Education will be held at the Yogyakarta State University in 2011.

Starting in 2009 IndoMS organize International Conferences. IICMA2009 is IndoMS International Conference on Mathematics and its Applications 2009. It is majority supported by Directorate General of Higher Educations (DGHE), Department of National Education, Indonesia through “Professional Organization Symposium Competition Program” (Program Hibah Symposium Organisasi Profesi). IndoMS is one of professional organizations which granted by this program. In this conference we facilitate researchers and users of mathematics to exchange ideas and discus research results and development of mathematics internationally in the fields of mathematics including mathematics education and its applications.

All 166 full papers in the conference has been reviewed by 36 competence experts. As results of the conference, we got 20 papers are feasible to be published in international journals, 17 papers are feasible to be published in aspirated international journals, 32 papers are feasible to be published in national journals, 65 papers published in this proceeding, and 32 papers are rejected

Finally, I would like to express my sincere appreciation to:

- Dean of Faculty of Mathematics and natural Sciences and Rector of Gadjah Mada University for the permission and cooperation in holding the Conference
- Steering Committee and Organizing Committee for all efforts for the success of the conference

- All invited speakers and all participants from Indonesia and abroad for the active participation of the conference.
- All editors for all efforts to finish this proceeding.

Last but not least, my sincere appreciation is also extended to the DGHE for the major support of the conference.

Yogyakarta, January, 2010
President of IndoMS 2008-2010

Prof. Dr.rer.nat. Widodo

Preface from the Committee

The proceeding of IICMA Conference is a collection of all selected papers that were presented in IndoMS International Conference on Mathematics and its Applications (IICMA) 2009 held at Department of Mathematics – Gadjah Mada University, Yogyakarta, October 12th – 13th, 2009. The selected papers are based on the reviewed results by 36 competence reviewers. Each paper has been reviewed by at least two reviewers. On behalf of the Committee, we would like to say thank you very much to all of the reviewers.

There are 118 papers in this proceeding coming from diverse aspects of mathematics ranging from Analysis, Applied Mathematics, Algebra, Theoretical Computer Science, Mathematics Education, and other related topics. We are sure the papers will inspire, not only writers, but also many other researches in developing mathematics and its applications. Please find the benefit of the proceeding.

Yogyakarta, January 2010
On behalf of the Committee IICMA 2009

Dr. Ch. Rini Indrati, M.Si.
Chair

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A MAX-PLUS ALGEBRA APPROACH TO CRITICAL PATH ANALYSIS IN THE PROJECT NETWORK WITH FUZZY ACTIVITY TIMES

M. ANDY RUDHITO, SRI WAHYUNI, ARI SUPARWANTO, AND F. SUSILO

Abstract. The activity times in a project network are seldom precisely known, and then could be represented into the fuzzy number, that is called fuzzy activity times. This paper aims to determine the fuzzy earliest starting times, fuzzy total duration time and fuzzy critical path using max-plus algebra approach. The finding shows that the project network with fuzzy activity times can be represented as a matrix over fuzzy number max-plus algebra. The project network dynamics can be represented as a system of fuzzy number max-plus linear equations. From the solutions of the system we can determine the fuzzy earliest starting times and fuzzy total duration time. The fuzzy critical path with a certain degree of critically can be determine through an interval critical path determination. We know that an alpha-cut of the fuzzy activity times is an interval activity time. The interval critical path can be determined through a crisp critical path determination. Meanwhile, the crisp critical path can be determined via a computation using max-plus algebra approach.

Key words and Phrases : max-plus algebra, project network, fuzzy activity times, fuzzy critical path.

1. Introduction

Let $\mathbf{R}_\varepsilon := \mathbf{R} \cup \{\varepsilon\}$ with $\mathbf{R}$ thr set of all real numbers and $\varepsilon : = -\infty$. In $\mathbf{R}_\varepsilon$ defined two operations : $\forall a,b \in \mathbf{R}_\varepsilon$, $a \oplus b := \max(a, b)$ and $a \otimes b : = a + b$. We can show that ($\mathbf{R}_\varepsilon$, $\oplus$, $\otimes$) is a commutative idempotent semiring with neutral element $\varepsilon = -\infty$ and unity element $e = 0$. Moreover, ($\mathbf{R}_\varepsilon$, $\oplus$, $\otimes$) is a semifield, that is ($\mathbf{R}_\varepsilon$, $\oplus$, $\otimes$) is a commutative semiring, where for every $a \in \mathbf{R}$ there exist $-a$ such that $a \otimes (-a) = 0$. Thus, ($\mathbf{R}_\varepsilon$, $\oplus$, $\otimes$) is a *max-plus algebra*, and is written as $\mathbf{R}_{\max}$. One can define $x^{\otimes 0} := 0$, $x^{\otimes k} := x \otimes x^{\otimes k-1}$, $\varepsilon^{\otimes 0} : = 0$ and $\varepsilon^{\otimes k} : = \varepsilon$, for $k = 1, 2, ...$

The operations $\oplus$ and $\otimes$ in $\mathbf{R}_{\max}$ can be extend to the matrices operations in $\mathbf{R}_{\max}^{m\times n}$, with $\mathbf{R}_{\max}^{m\times n} : = \{A = (A_{ij}) \mid A_{ij} \in \mathbf{R}_{\max}$, for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n\}$, the set of all matrices over max-plus algebra. Specifically, for $A, B \in \mathbf{R}_{\max}^{n\times n}$ we define $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$ and $(A \otimes B)_{ij} = \bigoplus_{k=1}^{n} A_{ik} \otimes B_{kj}$. We also define matrix

$$E \in \mathbf{R}_{\max}^{n\times n},\ (E)_{ij} : = \begin{cases} 0\ , & \text{if } i = j \\ \varepsilon\ , & \text{if } i \neq j \end{cases} \text{ and } \mathcal{E} \in \mathbf{R}_{\max}^{m\times n},\ (\varepsilon)_{ij} := \varepsilon \text{ for every } i \text{ and } j .$$

For any matrices $A \in \mathbf{R}_{\max}^{nxn}$, one can define $A^{\otimes 0} = E_n$ and $A^{\otimes k} = A \otimes A^{\otimes k-1}$ for k = 1, 2, ... For any weighted, directed graph $\mathcal{G} = (V, A)$ we can define a matrix $A \in \mathbf{R}_{\max}^{n\times n}$, $A_{ij} = \begin{cases} w(j,i), & \text{if } (j,i) \in \mathcal{A} \\ \varepsilon, & \text{if } (j,i) \notin \mathcal{A}. \end{cases}$, called the *weight-matrix* of graph $\mathcal{G}$. Further details about max-plus algebra, matrix and graph can be found in Baccelli *et.al* (2001) and Rudhito A (2003).

A method to analyze the critical path in the project network with crisp (real) activity times, using max-plus algebra approach had been developed in Rudhito, *et.al.* (2008[b]). The followings are some result in brief. A *project network S* with crisp activity times, is a directed, strongly connected, acyclic, crisp weighted graph $S = (\mathcal{V}, \mathcal{A})$, with $V = \{1, 2, , ... , n\}$ suct that if $(i, j) \in \mathcal{A}$, then $i < j$. Let x_i^e is be *crisp-earliest starting time* from node i and $\mathbf{x^e} = [\, x_1^e, x_2^e, \ldots, x_n^e\,]^{\mathrm{T}}$. For a project network with crisp activity times, with n nodes and A the weight matrix of graph of the networks, then

$$\mathbf{x^e} = (E \oplus A \oplus \ldots \oplus A^{\otimes n-1}) \otimes \boldsymbol{b^e} = A^* \otimes \boldsymbol{b^e} \qquad (1)$$

with $\boldsymbol{b^e} = [0, \varepsilon, \ldots, \varepsilon]^{\mathrm{T}}$. Furthermore x_n^e is the total crisp duration time of the project. Let x_i^l is be *crisp latest completion time* for all activities that come to node i and $\mathbf{x}^l = [\, x_1^l, x_2^l \ldots., x_n^l\,]$. For the project network above, vector

$$\mathbf{x}^l = -(\,(A^{\mathrm{T}})^* \otimes \boldsymbol{b}^l\,) \qquad (2)$$

with $\boldsymbol{b^l} = [\varepsilon, \varepsilon, \ldots, -x_n^e\,]^{\mathrm{T}}$. From (1), (2) and weight-matrix A, we can determine the *total slack times* for activities $(i, j) \in \mathcal{A}$

$$TS_{ij} = x_j^l - x_i^e - A_{ij}. \qquad (3)$$

Definition 1.1 *An activity $(i, j) \in \mathcal{A}$ is called a* ***crisp-critical activity*** *if $TS_{ij} = 0$. A path $p \in P$ is called* ***crisp-critical path*** *if all activities belonging to p are critical activities, where P is the set of all path in network from node 1 to node n.*

An approach to analyze the critical path in the project network with interval activity times had been developed in Chanas & Zielinski (2001) with the classical approach. Meanwhile, we will develop on the max-plus algebra approach. We will use some results in Chanas & Zielinski (2001). The followings are some result in brief. A *project network* $\boldsymbol{S}$ with interval activity times, is a directed, strongly connected, acyclic, interval weighted graph $\boldsymbol{S} = (\mathcal{V}, \boldsymbol{\mathcal{A}})$, with $V = \{1, 2, , ... , n\}$ such that, if $(i, j) \in \boldsymbol{\mathcal{A}}$, then $i < j$.

Definition 1.2 *A path $p \in P$ is called an* ***interval-critical path*** *in* $\boldsymbol{S}$ *if there exist a set of times A_{ij}, $A_{ij} \in [\,\underline{A}_{ij}, \overline{A}_{ij}\,]$, $(i, j) \in \boldsymbol{\mathcal{A}}$, such that p is crisp-critical, after replacing the interval times* A_{ij} *with crisp activity time A_{ij}, $(i, j) \in \boldsymbol{\mathcal{A}}$.*

Theorem 1.2 [3] *A path $p \in P$ is interval-critical in* $\boldsymbol{S}$ *if and only if crisp-critical in S, in which the interval activity times* $\mathrm{A}_{ij} \in [\,\underline{A}_{ij}, \overline{A}_{ij}\,]$, $(i, j) \in \boldsymbol{\mathcal{A}}$, *have been replace* *with crisp activity times A_{ij} determined by means of the following formula*

$$A_{ij} = \begin{cases} \overline{A}_{ij}, \text{if } (i, j) \in p, \\ \underline{A}_{ij}, \text{if } (i, j) \notin p. \end{cases} \tag{4}$$

We will discuss the determining the fuzzy earliest starting times, fuzzy total duration time and fuzzy critical path using max-plus algebra approach. Basically, the notion of the determining fuzzy earliest starting times, fuzzy total duration time is analog with the method in Rudhito, *et.al.* (2008[b]), where the activity times is crisp. We will develop for the fuzzy activity times. We replace the deterministic activity times with the fuzzy activity times. The discussion on the fuzzy critical path using will be based on the result in Rudhito, *et.al.* (2008[b]) and Chanas & Zielinski (2001, 2002) like above. We also give some examples for illustration.

We will used some concepts and result on the fuzzy set and fuzzy number, that can be found in Zimmermann, H.J., (1991), Lee, K.H. (2005) and Susilo, F. (2006), fuzzy number max-plus algebra and matrix over fuzzy max-plus algebra in Rudhito, *et.al.* (2008[a]) and iterative system of fuzzy number max-plus linear equations in Rudhito, *et.al.* (2008[c])

2. Main Results

Definition 2.1 A *project networks with fuzzy activity times* $\widetilde{\mathbf{S}}$ *is a directed, connected, acyclic and fuzzy number-valued weighted graph* $\widetilde{\mathbf{S}} = (\mathcal{V}, \widetilde{\mathcal{A}})$, *with* $\mathcal{V}$= $\{1, 2, \ldots, n\}$ *such that, if* $(i, j) \in \widetilde{\mathcal{A}}$, *then* $i < j$.

In this project, an arch represent an *activity,* fuzzy number-valued weight of an arch represent an *activity times*, so an fuzzy number-valued weight in this networks is a nonnegative fuzzy number (where its α-cut is an nonnegative interval).

Let $\widetilde{x}_i^e$ be the fuzzy earliest starting time from node i.

$$\widetilde{A}_{ij} = \begin{cases} \text{fuzzy activity times from } j \text{ to } i, \text{if } (j,i) \in \widetilde{A} \\ \widetilde{\varepsilon}, \qquad\qquad\qquad \text{if } (j, i) \notin \widetilde{A}. \end{cases}$$

We assume that $\widetilde{x}_1^e = \widetilde{0}$ and with max-plus fuzzy number notation we have

$$\widetilde{x}_1^e = \begin{cases} \widetilde{0}, & \text{if } i = 1 \\ \widetilde{\bigoplus\limits_{1 \le j \le n}} (\widetilde{A}_{ij} \,\widetilde{\otimes}\, \widetilde{x}_j^e), & \text{if } i > 1. \end{cases} \tag{5}$$

Let $\widetilde{A}$ be the fuzzy number weight matrix of the fuzzy number-valued weighted graph of the networks, $\widetilde{\mathbf{x}}^e = [\widetilde{x}_1^e, \widetilde{x}_2^e, \ldots, \widetilde{x}_n^e]^{\mathrm{T}}$ and $\widetilde{\boldsymbol{b}}^e = [0, \widetilde{\varepsilon}, \ldots, \widetilde{\varepsilon}]^{\mathrm{T}}$, equation (5) can be written in an iterative system of fuzzy max-plus linear equations

$$\tilde{\boldsymbol{x}}^e = \tilde{A} \;\tilde{\otimes}\; \tilde{\boldsymbol{x}}^e \;\tilde{\oplus}\; \tilde{\boldsymbol{b}}^{\boldsymbol{e}}. \tag{6}$$

Since the project networks is acyclic directed graph, then there are no circuit, so according to the result in Rudhito, *et.al.* (2008[c]), $\tilde{A}$ is definite. And then also according to the result in Rudhito, *et.al.* (2008[c]), the fuzzy number vector $\tilde{\boldsymbol{x}}^e$ with $\tilde{x}_i^e = \bigcup_{\alpha\in[0,1]} \tilde{c}_i^{\alpha}$ where $\tilde{c}_i^{\alpha}$ is a fuzzy set in $\mathbf{R}$ with membership function $\mu_{\tilde{c}^{\alpha}}(x) = \alpha\,\chi_{(A^*\otimes \boldsymbol{b}^e)_i^{\alpha}}(x)$, where $\chi_{(A^*\otimes \boldsymbol{b}^e)_i^{\alpha}}$ is a characteristic function of set $(A^* \otimes \boldsymbol{b}^{\boldsymbol{e}})_i^{\alpha}$, is a unique solution of the system (6), that is the vector of the fuzzy earliest starting time for each node in the project .

Notice that $(\tilde{A}^*)_{n1}$ is a maximum weigh of path form the start node to the end node, hence $\tilde{x}_n^e$ is the fuzzy total duration time of the project. We summarize the description above in the Theorem 2.1.

Theorem 2.1 *Given a project network with fuzzy activity times, with n node and* $\tilde{A}$ *is the weight matrix of the fuzzy number-valued weighted graph of networks. The vector* $\tilde{\boldsymbol{x}}^e$ *is given by fuzzy number vector with* $\tilde{x}_i^e = \bigcup_{\alpha\in[0,1]} \tilde{c}_i^{\alpha}$ *where* $\tilde{c}_i^{\alpha}$ *is a fuzzy set in* $\mathbf{R}$ *with membership function* $\mu_{\tilde{c}^{\alpha}}(x) = \alpha\,\chi_{(A^*\otimes \boldsymbol{b}^e)_i^{\alpha}}(x)$ *and* $\chi_{(A^*\otimes \boldsymbol{b}^e)_i^{\alpha}}$ *is a characteristic function of set* $(A^* \otimes \boldsymbol{b}^{\boldsymbol{e}})_i^{\alpha}$, with $\tilde{\boldsymbol{b}}^{\boldsymbol{e}} = [0, \tilde{\varepsilon}, \ldots, \tilde{\varepsilon}]^{\mathrm{T}}$.

Proof: see description above. ■

Before we give an example, we recall about a special types of fuzzy number. A *triangular fuzzy number* $\tilde{a}$, which is written as TFN(a_1, a, a_2) or (a_1, a, a_2), is a fuzzy number with membership function

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a_1}{a-a_1}, & a_1 \le x \le a \\ \dfrac{a_2-x}{a_2-a}, & a \le x \le a_2 \\ 0, & others \end{cases}.$$

The *support of* $\tilde{a}$ is an open interval (a_1, a_2) and its α-cut is

$$a^{\alpha} = \begin{cases} [(a-a_1)\alpha + a_1, -(a_2-a)\alpha + a_2] & ,\alpha \in (0,1] \\ [a_1, a_2] & ,\alpha = 0 \end{cases}.$$

Example 2.1 *Consider the project network in Figure 2.1.*

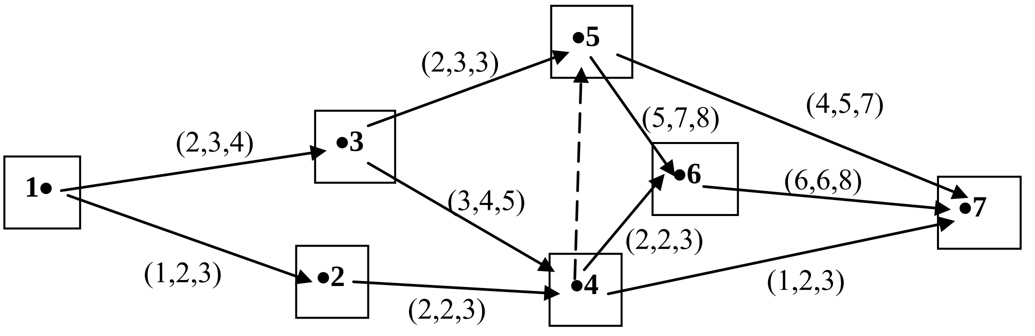

Figure 2.1 Project Network

We have

$$\widetilde{A} = \begin{bmatrix} \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} \\ (1,2,3) & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} \\ (2,3,4) & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} \\ \widetilde{\varepsilon} & (2,2,3) & (3,4,5) & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} \\ \widetilde{\varepsilon} & \widetilde{\varepsilon} & (2,3,3) & \widetilde{0} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} \\ \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & (2,2,3) & (5,7,8) & \widetilde{\varepsilon} & \widetilde{\varepsilon} \\ \widetilde{\varepsilon} & \widetilde{\varepsilon} & \widetilde{\varepsilon} & (1,2,3) & (4,5,7) & (6,6,8) & \widetilde{\varepsilon} \end{bmatrix}.$$

Using MATLAB computer program, we have bounds of α-cut of components of vector $\widetilde{\mathbf{x}}^e$ and the sketch of them, for $\alpha = 0, 0.05, \ldots, 0.95, 1$, is the Figure 2.2 bellow.

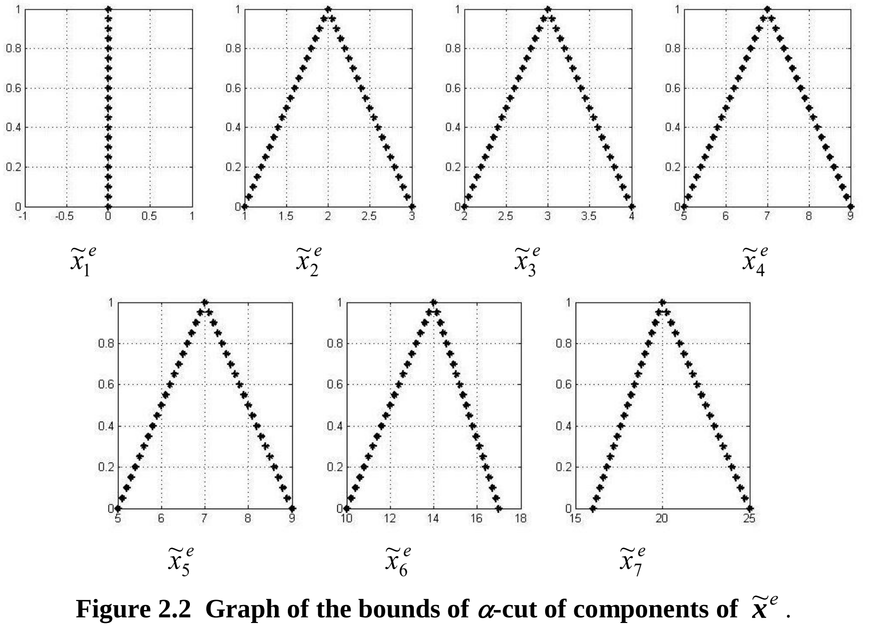

Figure 2.2 Graph of the bounds of α-cut of components of $\widetilde{\mathbf{x}}^e$.

From the Figure 1, we have $\tilde{x}_1^e$ = TFN(0, 0, 0), $\tilde{x}_2^e$ = TFN(1, 2, 3), $\tilde{x}_3^e$ = TFN(2, 3, 4), $\tilde{x}_4^e$ = TFN (5, 7, 9), $\tilde{x}_5^e$ = TFN (5, 7, 9), $\tilde{x}_6^e$ = TFN (10, 14, 17) and $\tilde{x}_7^e$ = TFN (16, 20, 25). The fuzzy total duration time of the project is $\tilde{x}_7^e$ = TFN(16, 20, 25).

In the fuzzy critical path discussion below, we will talk about path degree of criticality. We will review some definition and theorem, which are develop from Chanas & Zielinski (2001)

Definition 2.2 *The fuzzy set* $\tilde{P}$ *in set P with the membership function* $\mu_{\tilde{P}} : P \to [0,1]$ *determined by formula*

$$\mu_{\tilde{P}}(p) = \sup_{\substack{A_{ij} \in \mathbf{R}^+, (i,j) \in \tilde{A} \\ \text{and } p \text{ is crisp-critical with} \\ \text{crisp activity times} \\ \text{equal to } A_{ij}, (i,j) \in \tilde{A}}} \min_{(i,j) \in A} \mu_{\tilde{A}_{ij}}(A_{ij}), \; p \in P$$

is called ***fuzzy-critical path*** *in* $\tilde{\mathbf{S}}$ *with degree* $\mu_{\tilde{P}}(p)$.

Definition 2.3 *The scalar* $\alpha \in (0, 1]$ *is called* ***feasible under the path*** *p if p is interval-critical in the network* $\tilde{\mathbf{S}}$ *with interval activity times* $A_{ij} = A_{ij}^{\alpha}$, *where* A_{ij}^{α} *are* α*-cut of fuzzy activity times* $\tilde{A}_{ij}$.

Theorem 2.2 *The following equality holds :*
$\mu_{\tilde{P}}(p) = sup\{\ \alpha \mid \alpha$ *is fisible value under the path* $p \in P$ $\}$.

Proof: Obvious, according to the Decomposition Theorem in fuzzy set. ■

Definition 2.4 *A path p is said to have* ***degree of criticality*** $\mu_{\tilde{P}}(p) = 0$ *if* $\alpha = 0$ *is not feasible under the path p.*

Now we present an algorithm for computing the criticality degree $\mu_{\tilde{P}}(p)$ of path $p \in P$. This algorithm is developed from an algorithm in Chanas & Zielinski (2001). The algorithm is based on the idea bisection of the interval [0, 1] of possible values of α to compute the maximal feaseble $\alpha_{\max}$. For testing of feasibility, we used (4) and determining crisp critical path, we used (1), (2) and (3).

Algorithm 2.1 (*Determining the path degree of criticality***)**
Step 1 :
Assign k :=0.
Step 2 :
Tes feasibility $\alpha = 0$ under the path p. If it is not, then $\alpha_{\max} = 0$ and go to Step 6.
Step 3 :
Tes feasibility $\alpha_k = 1$ under the path p. If it is feasible under the path p, then $\alpha_{\max}$ = 1 and go to Step 6.
Step 4 :
Assign $k := k + 1$.

$$\alpha_k := \begin{cases} \alpha_{k-1} + \dfrac{1}{2^k}, & \text{if } \alpha_{k-1} \text{ feasible} \\ \alpha_{k-1} - \dfrac{1}{2^k}, & \text{if } \alpha_{k-1} \text{ not feasible.} \end{cases}$$

Tes feasibility α_k under the path p. If it is feasible assign $\alpha_{\max} = \alpha_k$.
Step 5:
If $k < K$, then go to step 4.
Step 6:
Assign $\mu_{\tilde{P}}(p) = \alpha_{\max}$. Stop.
With: $K \geq N / {}^{10}\log 2$, with absolute error of computation 10^{-N}.

Example 2.2 *Consider the project network in Example 2.1. Applying Algorithm 2.1 we have obtained results which are listed in Table 2.1. The path degree of criticality have been computed with accuracy 10^{-2}.*

Table 2.1 The path degree of criticality

No	Path p	$\mu_{\tilde{P}}(p)$
1	1→3→5→7	0
2	1→3→5→6→7	0,0039
3	1→3→4→5→7	0
4	1→3→4→5→6→7	1
5	1→3→4→6→7	0
6	1→3→4→7	0
7	1→2→4→5→7	0
8	1→2→4→5→6→7	0,25
9	1→2→4→6→7	0
10	1→2→4→7	0

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